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Supersymmetric Higgs boson decays in the MSSM with explicit CP violation

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Abstract. Decays into neutralinos and charginos are among the most accessible supersymmetric decay modes of Higgs particles in most supersymmetric extensions of the standard model. In the presence of explicitly CP violating phases in the soft breaking sector of the theory, the couplings of Higgs bosons to charginos and neutralinos are in general complex. Based on a specific benchmark scenario of CP violation, we analyze the phenomenological impact of explicit CP violation in the minimal supersymmetric standard model on these Higgs boson decays. The presence of CP violating phases could be confirmed either directly through the measurement of a CP odd polarization asymmetry of the produced charginos and neutralinos, or through the dependence of CP even quantities (branching ratios and masses) on these phases.

The experimental observation of Higgs particles is crucial for our understanding of electroweak symmetry breaking. Thus the search for Higgs bosons is one of the main goals of future colliders such as the Large Hadron Collider (LHC) and high energy e^+e^- linear colliders (LC). Once a Higgs boson is found, it will be of the utmost importance to perform a detailed investigation of its properties so as to establish the Higgs mechanism as the basic way to generate the masses of the known particles. To this end, precise theoretical predictions for the main decay channels as well as the production cross sections are essential.

In the minimal supersymmetric standard model (MSSM), CP violating phases of some dimensionful parameters (most of which parameterize the soft breaking of supersymmetry) cause the CP even and CP odd neutral Higgs bosons to mix via loop corrections [1,2]; the most important contribution usually comes from the top–stop sector. The loop-induced CP violation in the MSSM Higgs sector can by itself be large enough to affect the Higgs phenomenology significantly at present and future colliders [1,3–6]. Moreover, these CP phases can also lead to "direct" CP violation in the couplings of Higgs bosons to superparticles [6]. The impact of such potentially large CP violating effects on Higgs boson decays has recently been studied in [5], where the dominant decays of the charged and neutral Higgs bosons, into standard model (SM) particles and squark pairs, were investigated in the context of the MSSM with explicit CP violation. In this note, we extend these analyses by including the potentially significant decays of Higgs particles into neutralinos and charginos. These decays have been studied in detail in the CP invariant version of the MSSM in [7,8]. We allow for CP violation both through loop effects in the Higgs sector, using a form of the Higgs mass matrix that is applicable for all combinations of stop mass parameters [2], and through phases in the chargino and neutralino mass matrices. We find that the CP phases can significantly alter the branching ratios for these decays; moreover, they can also lead to the appearance of large CP odd polarization asymmetries.

As is well known [7,8], Higgs boson decays to neutralinos and charginos,

$$H_k^0 \to \tilde{\chi}_i^0 \tilde{\chi}_j^0, \quad \tilde{\chi}_i^+ \tilde{\chi}_j^- \quad \text{and} \quad H^\pm \to \tilde{\chi}_i^0 \tilde{\chi}_j^\pm, \tag{1}$$

could play a potentially important role. Here $k = 1,2,3$ labels the three neutral Higgs bosons of the MSSM, while $i,j =$ 1–4 and 1, 2 for neutralinos and charginos, respectively. If R-parity is conserved and $\tilde{\chi}_1^0$ is the lightest supersymmetric stable particle (LSP), the $\tilde{\chi}_1^0\tilde{\chi}_1^0$ final states are invisible. The other $\tilde{\chi}\tilde{\chi}$ modes would also be accompanied by a large amount of missing energy coming from the $\tilde{\chi}_i^0$ and $\tilde{\chi}_i^\pm$ decay cascades, which lead to (at least) two LSPs per Higgs boson decay. If kinematically allowed, the branching ratios for some of the supersymmetric Higgs boson decay modes (1) will be large, unless the ratio of vacuum expectation values (VEVs) $\tan\beta \equiv \langle h_2^0\rangle/\langle h_1^0\rangle \gg 1$; here h_2 (h_1) is the Higgs doublet coupling to top (bottom) quarks. If $\tan\beta$ is very large, the b and τ Yukawa couplings become large, in which case the modes (1) will be subdom-

inant. We will therefore focus on a scenario with moderate $\tan\beta$.

In order to determine the masses of charginos and neutralinos as well as their couplings to Higgs particles, we have to specify the higgsino mass parameter μ and the $U(1)$ and $SU(2)$ gaugino mass parameters M_1 and M_2. Following the notation of [9], we write the chargino matrix as

$$\mathcal{M}_C = \begin{pmatrix} M_2 & \sqrt{2}m_W c_\beta \\ \sqrt{2}m_W s_\beta & \mu \end{pmatrix}. \tag{2}$$

Diagonalizing this matrix with the help of two unitary matrices $U_{\rm R}, U_{\rm L}$, i.e., $\mathcal{M}_{C,\rm diag} = U_{\rm R}\mathcal{M}_C U_{\rm L}^\dagger$, generates the light and heavy chargino states $\tilde{\chi}_i^\pm$ $(i = 1, 2)$. Similarly, the neutralino mass matrix

$$\mathcal{M}_N = \begin{pmatrix} M_1 & 0 & -m_Z c_\beta s_{\rm W} & m_Z s_\beta s_{\rm W} \\ 0 & M_2 & m_Z c_\beta c_{\rm W} & -m_Z s_\beta c_{\rm W} \\ -m_Z c_\beta s_{\rm W} & m_Z c_\beta c_{\rm W} & 0 & -\mu \\ m_Z s_\beta s_{\rm W} & -m_Z s_\beta c_{\rm W} & -\mu & 0 \end{pmatrix} \tag{3}$$

is diagonalized by the unitary matrix N, $\mathcal{M}_{N,\rm diag} = N^* \mathcal{M}_N N^\dagger$, leading to four neutralino states $\tilde{\chi}_i^0$ $(i = 1, 2, 3, 4)$, ordered with rising mass. In (2) and (3) we have used $s_\beta \equiv \sin\beta$, $c_\beta \equiv \cos\beta$, and $s_{\rm W}, c_{\rm W}$ are the sine and cosine of the electroweak mixing angle. In CP noninvariant theories, all mass parameters can be complex. However, one can always find a field basis where the $SU(2)$ mass parameter M_2 as well as the VEVs are real and positive. The $U(1)$ mass parameter M_1 is then assigned the phase Φ_1, and the higgsino mass parameter μ has the phase Φ_μ. We will adopt this convention in this paper.

The couplings of Higgs bosons to charginos and neutralinos are determined by the unitary matrices $U_{\rm L,R}$ and N defined above, as well as by the orthogonal matrix O relating the weak eigenstates $\varphi_k \equiv \{a, \phi_1, \phi_2\}$ to the three neutral Higgs boson mass eigenstates H_k^0 $(k = 1, 2, 3)$, $H = O^{\rm T}\varphi$ [1,2]; here $\phi_i = 2^{1/2}{\rm Re}h_i^0$ and $a = 2^{1/2}(s_\beta {\rm Im}h_1^0 + c_\beta {\rm Im}h_2^0)$. Specifically, the vertices relevant for the decays of neutral Higgs bosons are given by

$$\begin{aligned} &\langle \tilde{\chi}_{i\rm R}^- | H_k^0 | \tilde{\chi}_{j\rm L}^- \rangle \equiv g X_{k;ij}^{\rm L} \\ &= -\frac{g}{\sqrt{2}} \left(U_{{\rm R}i1} U_{{\rm L}j2}^* G_{2k} + U_{{\rm R}i2} U_{{\rm L}j1}^* G_{3k} \right), \\ &\langle \tilde{\chi}_{i\rm R}^0 | H_k^0 | \tilde{\chi}_{j\rm L}^0 \rangle \equiv \frac{g}{2} Y_{k;ij}^{\rm L} \\ &= -\frac{g}{4} \left[(N_{i3}^* G_{2k} - N_{i4}^* G_{3k})(N_{j2}^* - t_W N_{j1}^*) + (i \leftrightarrow j) \right], \end{aligned} \tag{4}$$

where we have defined the complex coefficients $G_{2k} = O_{2k} - {\rm i}s_\beta O_{1k}$ and $G_{3k} = O_{3k} - {\rm i}c_\beta O_{1k}$. The corresponding couplings for right-handed charginos and neutralinos are given by

$$X_{k;ij}^{\rm R} = X_{k;ji}^{\rm L*}, \quad Y_{k;ij}^{\rm R} = Y_{k;ji}^{\rm L*}. \tag{5}$$

Similarly, the relevant charged-Higgs–neutralino–chargino vertices are

$$\begin{aligned} &\langle \tilde{\chi}_{i\rm R}^0 | H^+ | \tilde{\chi}_{j\rm L}^- \rangle \equiv g Z_{ij}^{\rm L} \\ &= -\frac{g s_\beta}{\sqrt{2}} \left[\sqrt{2} N_{i3}^* U_{{\rm L}j1}^* - (N_{i2}^* + t_W N_{i1}^*) U_{{\rm L}j2}^* \right], \\ &\langle \tilde{\chi}_{i\rm L}^0 | H^+ | \tilde{\chi}_{j\rm R}^- \rangle \equiv g Z_{ij}^{\rm R} \\ &= -\frac{g c_\beta}{\sqrt{2}} \left[\sqrt{2} N_{i4} U_{{\rm R}j1}^* + (N_{i2} + t_W N_{i1}) U_{{\rm R}j2}^* \right]. \end{aligned} \tag{6}$$

The couplings of (4) and (6) show that all the Higgs particles couple to one gaugino and one higgsino component of charginos and neutralinos. This is not surprising, since these interactions result from the supersymmetric Higgs boson–gaugino–higgsino interactions in the basic supersymmetric Lagrangian written in terms of current eigenstates. In particular, in the limit of vanishing higgsino–gaugino mixing, i.e., $|\mu| \to \infty$ or $|M_{1,2}| \to \infty$, all diagonal $H\tilde{\chi}_i\tilde{\chi}_i$ couplings vanish at the tree–level[1]. On the other hand, when $|\mu| \sim |M_1|$ or $|\mu| \sim |M_2|$, gaugino–higgsino mixing will be sizable and the $H\tilde{\chi}_i\tilde{\chi}_i$ couplings can be significant. Moreover, the total decay width for Higgs boson decays into charginos and neutralinos will remain large even for small higgsino–gaugino mixing, if the Higgs mass in question exceeds $|M_2| + |\mu|$ and $|M_1| + |\mu|$, so that decays into one gaugino-like and one higgsino-like state are allowed.

CP violation in the couplings of neutral Higgs bosons to CP self-conjugate final states is signaled by the simultaneous existence of scalar and pseudoscalar components, which happens if, e.g., neither $X^{\rm R} = X^{\rm L}$ nor $X^{\rm R} = -X^{\rm L}$. Equation (5) therefore implies that CP will be violated unless the couplings X, Y are either purely real or purely imaginary. From (4) we see that such a nontrivial phase in the couplings results if either the mixing matrices $U_{\rm L}, U_{\rm R}, N$ are complex, due to CP violation in the chargino and neutralino sector, or if O_{1k} and $O_{(2,3)k}$ are simultaneously nonzero for some Higgs boson H_k^0. The latter signals "indirect" CP violation through mixing between scalar and pseudoscalar Higgs fields. In the MSSM this mixing is predominantly induced by loops involving top squarks, and is quantified by the dimensionless parameter [2]

$$\Delta_{\tilde{t}} = \frac{{\rm Im}(A_t \mu)}{m_{\tilde{t}_2}^2 - m_{\tilde{t}_1}^2}. \tag{7}$$

This mixing will be large only if ${\rm Im}(A_t\mu)$ is comparable to the squared top-squark masses[2]. Finally, the contributions from the top (s)quark sector to the CP even Higgs boson masses depend on the magnitude of the top-squark mixing parameter $X_t = -m_t(A_t + \mu^*/\tan\beta)$ as well as on the soft breaking top-squark mass parameters, $m_{\tilde{Q}}$ and $m_{\tilde{t}}$. So, at the one-loop level the Higgs boson masses (in particular, m_{H_1}) will depend significantly on the rephasing invariant

[1] Under certain circumstances, non-negligible diagonal couplings can result even in the absence of gaugino–higgsino mixing once loop corrections have been included [10]

[2] Equation (7) seems to imply that ${\rm Im}(A_t\mu)$ only has to be comparable to the *differences* of the squared stop masses. However, the mixing between (nearly) degenerate states, while apparently large, has no physical effect

phase $\Phi \equiv \arg(A_t\mu)$ only when $|A_t|$ and $|\mu|/\tan\beta$ are comparable in size. However, our treatment [2] includes leading two-loop corrections by using appropriately one-loop corrected top quark masses in the loop corrections to the Higgs boson masses. The gluino–stop loop corrections to m_t introduce some dependence of the Higgs boson masses on CP violating phases even if $|A_t| \gg |\mu|$. Our calculation thus includes pure Yukawa and mixed electroweak gauge–Yukawa corrections to one-loop order exactly (using the effective potential method), as well as leading (SUSY) QCD two-loop corrections. However, we do not include purely electroweak loop corrections to the Higgs masses [11].

Motivated by the above observations and experimental constraints on the lightest Higgs boson mass and on the light chargino mass [12], we consider the following benchmark scenario of SUSY parameters[3]:

$$\begin{aligned} &\tan\beta = 5, \quad M_A = 0.3\,\text{TeV}, \quad m_{\tilde{Q}} = m_{\tilde{t}} = 0.5\,\text{TeV}, \\ &|A_t| = 1.2\,\text{TeV}, \quad |\mu| = 250\,\text{GeV}, \quad |M_1| = 50\,\text{GeV}, \\ &M_2 = 150\,\text{GeV}, \quad |M_3| = 0.5\,\text{TeV}, \quad \arg(M_3) = 0. \end{aligned} \tag{8}$$

Here M_A is the RG-invariant one-loop improved pseudoscalar mass parameter, which sets the scale for the masses of the heavy MSSM Higgs bosons. Our choice $M_A = 0.3\,\text{TeV}$ implies that all these Higgs bosons will be accessible to the second stage of future LCs as currently planned, which will reach cms energy $s^{1/2} \sim 0.8$ to 1.2 TeV. Since in many SUSY models squark and Higgs boson masses are correlated, we also took relatively modest values for the soft breaking masses $m_{\tilde{Q}}, m_{\tilde{t}}$ of $SU(2)$ doublet and singlet stops, respectively. One then needs $|A_t| \sim 6^{1/2} m_{\tilde{Q}}$ (the so-called maximal stop mixing scenario) in order to safely satisfy the experimental lower bound on M_{H_1}; note that H_1^0 behaves similar to the SM Higgs boson in our case. A large $|A_t|$ also tends to maximize the CP violating mixing in the Higgs sector. Our choice of gaugino and higgsino mass parameters ensures significant mixing between $SU(2)$ gauginos and higgsinos. Furthermore, the gaugino masses are sufficiently small that the first two neutralinos and the lighter charginos are accessible to the decays of the heavy Higgs bosons, while H_1^0 can at least decay into two LSPs; note that the branching ratio for this last decay can be sizable only if the gaugino mass "unification condition" $M_1 \simeq M_2/2$ is violated [14]. On the other hand, Higgs boson decays into one gaugino-like and one higgsino-like state, which have the potentially largest branching ratios of all decays (1), are *not* allowed kinematically in our scenario. Moreover, our value of $\tan\beta$ is neither very large nor very small. We therefore consider our choice of parameters to be quite representative of general MSSM scenarios.

We have not yet fixed the values of most CP violating phases. There are important constraints on these phases in the MSSM, from experimental limits on the electric dipole moments (EDM) of the electron, neutron and ^{199}Hg [15]. However, these constraints can be avoided if there are cancellations between different supersymmetric diagrams and/or between different CP violating operators. Furthermore, since the constraints apply essentially only to the first- and possibly second-generation of matter fermions, they may be more relaxed for the third-generation coupling A_t, if we do not impose the assumption of universality between different generations. Large phases of μ and M_1 are also allowed if first- and second-generation sfermions are much heavier than sfermions of the third generation. We will therefore consider the entire range of Φ, Φ_μ and Φ_1 between 0 and π.

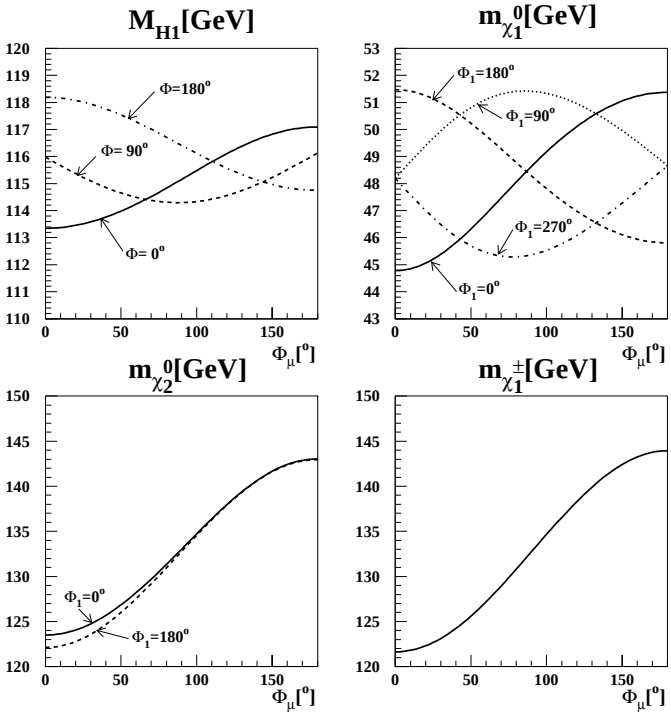

Fig. 1. The lightest Higgs boson mass, the lightest neutralino mass, the second lightest neutralino mass and the lighter chargino mass as functions of the phase Φ_μ for various values of $\Phi \equiv \arg(A_t\mu)$ and Φ_1

Figure 1 shows the dependence of the lightest Higgs boson mass, the two light neutralino masses and the light chargino mass on the phase Φ_μ for various values of the CP violating phases Φ and Φ_1. Once gluino–stop loop corrections to m_t are included, the neutral Higgs boson masses depend on both Φ and Φ_μ; indeed, since $|\mu|\cot\beta \ll |A_t|$ in our scenario, the phase dependence of m_{H_1} comes almost entirely from these corrections. Since they contribute to the Higgs masses only at two-loop order, the maximal variation of M_{H_1} with respect to both Φ and Φ_μ is less than 5 GeV, $\sim 5\%$ of the Higgs mass itself. We find that the heavy Higgs boson masses (not shown) also remain almost constant with their values close to 300 GeV. The lightest neutralino mass shows a somewhat stronger dependence on both Φ_1 and Φ_μ as shown in the upper right frame of Fig. 1. We therefore expect the branching ratio of the invisible decay $H_1^0 \to \tilde{\chi}_1^0\tilde{\chi}_1^0$ to be more sensitive to Φ_μ and Φ_1 than to Φ. Moreover, in spite of the

[3] A phase of the $SU(3)$ gaugino mass parameter M_3 could modify the top- and bottom-quark Yukawa couplings at the one-loop level, and could thus affect the branching ratios of the supersymmetric decays [13]

large value of $|A_t|$ we found that CP violating mixing between the heavy neutral Higgs bosons amounts to at most a few percent; this is due to the relatively small value of $|\mu|$. We therefore simply take $\Phi = 0$ in the following. The lower frames in Fig. 1 show that the approximate equality of $m_{\tilde{\chi}^0_2}$ and $m_{\tilde{\chi}^\pm_1}$ is maintained even in the presence of CP violation. $m_{\tilde{\chi}^\pm_1}$ is manifestly independent of Φ_1, but the Φ_1 dependence of $m_{\tilde{\chi}^0_2}$ is also essentially negligible. However, both masses depend quite strongly on Φ_μ. For our choice of parameters, both $\tilde{\chi}^0_2$ and $\tilde{\chi}^\pm_1$ are dominantly $SU(2)$ gauginos, with significant higgsino admixtures.

Since charginos and neutralinos are spin-1/2 particles, spin correlations of the $\tilde{\chi}\tilde{\chi}$ pair in the decay $H \to \tilde{\chi}\tilde{\chi}$ may allow us to probe CP violation in supersymmetric Higgs boson decays directly. The $\tilde{\chi}$ momenta cannot be identified event by event, due to the presence of invisible lightest neutralinos in the final state. Nevertheless, correlations may be estimated by statistically studying visible decay products from the spin-correlated $\tilde{\chi}\tilde{\chi}$ pairs. Including spin correlations in the final state, we can write the general form of the spin-correlated widths of supersymmetric Higgs boson decays into neutralino and chargino pairs in the following compact form:

$$\Gamma(\vec{P}^i, \vec{P}^j) = \frac{g^2 M_{H_k} \lambda^{1/2}}{16\pi S_{ij}} \Big\{ C_0^{ij}(1 + P_{\rm L}^i P_{\rm L}^j) + C_1^{ij}(P_{\rm L}^i + P_{\rm L}^j) + P_{\rm T}^i P_{\rm T}^j \left[C_2^{ij} \cos\phi_{ij} + C_3^{ij} \sin\phi_{ij} \right] \Big\} . \quad (9)$$

Here $P_{\rm L}^{i,j}$ and $P_{\rm T}^{i,j}$ are the degrees of longitudinal and transverse polarization of the final charginos or neutralinos, $\tilde{\chi}_i$ and $\tilde{\chi}_j$, respectively; $S_{ij} = 1$ unless the final state consists of two identical (Majorana) neutralinos in which case $S_{ii} = 2$; and $\lambda = (1 - \mu_{ik}^2 - \mu_{jk}^2)^2 - 4\mu_{ik}^2\mu_{jk}^2$ with $\mu_{ik}^2 = m_{\tilde{\chi}_i}^2/m_{H_k}^2$ is the usual two-body phase space function. Figure 2 shows a schematic description of the polarization configuration. The coefficients C_i $(i = 0, 1, 2, 3)$ in (9) are given by

$$\begin{aligned} C_0^{ij} &= (1 - \mu_{ik}^2 - \mu_{jk}^2)\left(|Q_{k;ij}^{\rm L}|^2 + |Q_{k;ij}^{\rm R}|^2\right) \\ &\quad - 4\mu_{ik}\mu_{jk}{\rm Re}(Q_{k;ij}^{\rm L} Q_{k;ij}^{\rm R*}), \\ C_1^{ij} &= \lambda^{1/2}\left(|Q_{k;ij}^{\rm L}|^2 - |Q_{k;ij}^{\rm R}|^2\right), \\ C_2^{ij} &= 2(1 - \mu_{ik}^2 - \mu_{jk}^2){\rm Re}\left(Q_{k;ij}^{\rm L} Q_{k;ij}^{\rm R*}\right) \\ &\quad - 2\mu_{ik}\mu_{jk}\left(|Q_{k;ij}^{\rm L}|^2 + |Q_{k;ij}^{\rm R}|^2\right), \\ C_3^{ij} &= -2\lambda^{1/2}{\rm Im}\left(Q_{k;ij}^{\rm L} Q_{k;ij}^{\rm R*}\right). \end{aligned} \quad (10)$$

Here the general couplings Q stand for X, Y or Z from (4)–(6) for chargino–chargino, neutralino–neutralino or chargino–neutralino pairs, respectively.

Note that all terms in (9) that are proportional to $P_{\rm L}^{i,j}$ or $P_{\rm T}^{i,j}$ will vanish after summation over $\tilde{\chi}$ spins. The various branching ratios are therefore determined entirely by the C_0^{ij}. Equations (4)–(6) show that the couplings Q all result from adding two or more terms. This means that not only ${\rm Re}(Q^{\rm L}Q^{*\rm R})$ but also the absolute values $|Q^{\rm L}|, |Q^{\rm R}|$ are sensitive to the CP violating phases. The partial widths also depend on the CP violating phases through the masses of the Higgs bosons, charginos and neutralinos.

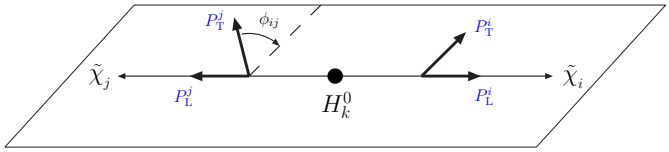

Fig. 2. Schematic description of the longitudinal and transverse polarization vectors $P_{\rm L}^{i,j}$ and $P_{\rm T}^{i,j}$, respectively, of the states $\tilde{\chi}_i$ and $\tilde{\chi}_j$. Here, ϕ_{ij} is the relative azimuthal angle of $P_{\rm T}^i$ with respect to $P_{\rm T}^j$

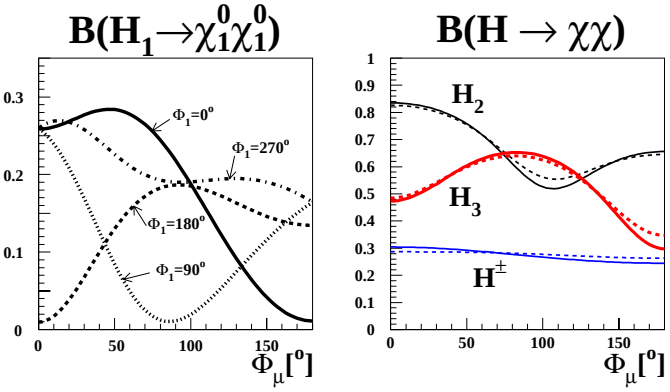

Fig. 3. The branching ratio of the lightest Higgs boson decay into the lightest neutralino pair as a function of Φ_μ for $\Phi = 0$ (i.e., $\Phi_{A_t} = -\Phi_\mu$), and $\Phi_1 = 0°, 90°, 180°$ and $270°$, respectively (*left frame*) and the sum of the branching ratios of the heavy Higgs bosons, $H^0_{2,3}$ and $H^\pm$ decays into all possible chargino and neutralino modes as a function of Φ_μ (*right frame*). In the right frame the *solid lines* are for $\Phi_1 = 0$ and the *dotted lines* for $\Phi_1 = 180°$

This is illustrated in Fig. 3. The left frame shows the branching ratio of the invisible decay $H_1^0 \to \tilde{\chi}_1^0\tilde{\chi}_1^0$. Such decays can be detected quite straightforwardly at e^+e^- colliders by measuring the missing mass in ZH_1^0 events [16]. It has recently been argued that a measurement of this invisible branching ratio of H_1^0 with an accuracy of a few percent should also be possible at the LHC, using H_1^0 produced in WW and ZZ fusion [17]. We see that this decay rate is very sensitive to Φ_1 and Φ_μ. This is partly due to the dependence of the lightest neutralino mass on the phases; see Fig. 1. Note that the phase space for this decay is quite small, so that relatively minor variations of the mass translate into large changes of the branching ratio. Moreover, this decay is P-wave suppressed, i.e., the partial width is $\propto \lambda^{3/2}$, if CP is conserved, but develops an S-wave piece in the presence of CP violation; for example, C_0^{11} in (10) vanishes at threshold ($\mu_{1k} = 0.5$) in the absence of CP violation. The branching ratio is therefore maximal at non-trivial values of Φ_μ and/or Φ_1. It is suppressed near $\Phi_\mu + \Phi_1 = 180°$ [mod 360°], where $m_{\tilde{\chi}_1^0}$ is maximal as shown in the upper right frame of Fig. 1.

The sum of the branching ratios for the heavy neutral and charged Higgs boson decays into all possible neu-

tralino and chargino modes is shown in the right frame of Fig. 3. In our case the four decay channels $\tilde{\chi}_1^0\tilde{\chi}_1^0$, $\tilde{\chi}_1^0\tilde{\chi}_2^0$, $\tilde{\chi}_2^0\tilde{\chi}_2^0$ and $\tilde{\chi}_1^+\tilde{\chi}_1^-$ are allowed for the neutral Higgs bosons $H_{2,3}^0$, while the two channels $\tilde{\chi}_1^-\tilde{\chi}_1^0$ and $\tilde{\chi}_1^-\tilde{\chi}_2^0$ are allowed for the charged Higgs boson H^-. At the LHC the dominant production process for the heavy neutral Higgs bosons is single production from gluon fusion, including production in association with a $b\bar{b}$ pair. It has been shown [18] that under favorable circumstances $H_{2,3}^0 \to \tilde{\chi}_2^0\tilde{\chi}_2^0$ decays can be detected at the LHC in the four lepton final state. However, this requires a large leptonic branching ratio for $\tilde{\chi}_2^0$, which in turn requires relatively light sleptons. We saw above that scenarios with light sleptons and CP violation are constrained severely by the electric dipole moment of the electron. Moreover, since the Higgs production cross sections at hadron colliders are uncertain even in the framework of the MSSM, it is not easy to translate a measurement of a number of events into a measurement of the corresponding branching ratio.

The dominant heavy Higgs production mechanisms at future e^+e^- colliders [16] are H^+H^- and $H_2^0H_3^0$ production[4]. The best search strategy is then probably to look for the decay of one of the heavy Higgs particles into third-generation fermions, while the other one is required to decay into $\tilde{\chi}$ states. We are not aware of a dedicated analysis of such final states, but the presence of an invariant mass peak for the third-generation fermion pair should allow one to extract this signal relatively cleanly. Alternatively one might simply measure the number of $b\bar{b}b\bar{b}$ and $b\bar{b}\tau^+\tau^-$ events with double invariant mass peak. Together with theoretical predictions for the total $H_2^0H_3^0$ production cross section, which in the MSSM essentially only depends on M_A once $M_A^2 \gg m_Z^2$, this would allow one to determine the heavy Higgs bosons' branching ratios into non-SM particles. This could be equated with the branching ratios for $H_i^0 \to \tilde{\chi}\tilde{\chi}$ decays if direct searches at the same experiment do not find other light sparticles into which the Higgs bosons might decay. We therefore expect the branching ratios to be measurable at future e^+e^- colliders with rather high accuracy; this should be true at least for the average of the H_2^0 and H_3^0 branching ratios, since it might be difficult to distinguish between these two Higgs bosons on an event by event basis. Finally, the heavy neutral Higgs bosons can also be produced singly as s-channel resonances at future $\mu^+\mu^-$ colliders [19].

We see that the summed branching ratios of the neutral heavy Higgs bosons are always quite large, varying between 30% and 80% depending on the value of Φ_μ. Since we have set $\Phi = 0$, H_2^0 is a pure CP odd state (often called A), while H_3^0 is purely CP even. Figure 1 shows that the phase space for the decays in question decreases monotonically as Φ_μ increases from 0 to 180°. Nevertheless the $H_2^0 \to \tilde{\chi}\tilde{\chi}$ branching ratio reaches a minimum for an intermediate value for Φ_μ. The reason is that the decay is now purely S-wave in the absence of CP violation, whereas nontrivial CP phases introduce a sizable P-wave component, which is strongly phase space suppressed in our case. For example, the $H_2^0\tilde{\chi}_1^+\tilde{\chi}_1^-$ coupling is almost purely scalar, rather than pseudoscalar, for $\Phi_\mu \simeq 100°$, near the minimum of $B(H_2^0 \to \tilde{\chi}\tilde{\chi})$. The branching ratio of H_3^0 decays shows essentially the opposite behavior, since H_3^0 is a CP even state; it can decay into an S-wave final state only in the presence of CP violation.

The $\tilde{\chi}_1^0\tilde{\chi}_1^0$ final state is subdominant in neutral Higgs boson decays; the larger phase space available for it is over-compensated by the small couplings to this bino-like neutralino. The couplings of H_2^0 to $\tilde{\chi}_1^+\tilde{\chi}_1^-$, $\tilde{\chi}_1^0\tilde{\chi}_2^0$ and $\tilde{\chi}_2^0\tilde{\chi}_2^0$ behave similarly, decreasing in magnitude with increasing Φ_μ; however, the corresponding couplings of the CP even state H_3^0, while again similar to each other, show the opposite dependence on Φ_μ. This can be traced back to the different decomposition of these two heavy Higgs bosons in terms of current eigenstates: $H_2^0 = 2^{1/2}\mathrm{Im}(s_\beta h_1^0 + c_\beta h_2^0)$, while for $M_A^2 \gg M_Z^2$, H_3^0 is approximately given by $2^{1/2}$ $\mathrm{Re}(s_\beta h_1^0 - c_\beta h_2^0)$. In contrast, H^- decays into neutralinos and charginos are dominated by the $\tilde{\chi}_1^-\tilde{\chi}_1^0$ final state, since the couplings $Z_{21}^{\mathrm{L,R}}$ of (6) are suppressed by large cancellations between the two terms in the square brackets. Note that the ratio of left– and right–handed $H^-\tilde{\chi}_1^-\tilde{\chi}_1^0$ couplings is proportional to $\tan\beta$. The charged Higgs boson decays therefore always have a large S-wave component, and are thus less sensitive to $\tilde{\chi}$ masses than neutral Higgs decays are; the $\tilde{\chi}$ mass dependence is reduced even further since the phase space for the $\tilde{\chi}_1^-\tilde{\chi}_1^0$ mode is anyway quite large. Furthermore, we find that the absolute value of the dominant coupling Z_{11}^{L} depends very little on the phases Φ_μ and Φ_1. The phase of this coupling does vary greatly, but this has little effect on the absolute value of the coefficient C_0 of (10), which determines the corresponding partial width, since $|Z_{11}^{\mathrm{R}}| \ll |Z_{11}^{\mathrm{L}}|$. Note finally that the branching ratio for the $\tilde{\chi}_1^-\tilde{\chi}_1^0$ mode is significant even though $H^- \to b\bar{t}$ decays are allowed. This indicates that the branching ratios for $H_{2,3}^0 \to \tilde{\chi}\tilde{\chi}$ can also be sizable even if $M_A > 2m_t$; recall that the partial widths for $H_{2,3}^0 \to \tilde{\chi}\tilde{\chi}$ decays will increase significantly if $M_A > |M_2| + |\mu|$.

In principle the spin-dependent terms in (9) allow more direct probes of CP violation. In case of neutral Higgs boson decays the C_0 and C_2 terms are even under a CP transformation while the C_1 and C_3 terms are odd. Moreover, the C_0, C_2 and C_3 terms are even under a $CP\widetilde{T}$ transformation, while the C_1 term is odd; here $\widetilde{T}$ describes "naive" time reversal, which flips the sign of all 3-momenta and spins but does not exchange the initial and final state. Note that a term can only be $CP\widetilde{T}$ odd but CPT even if it depends on some CP invariant, absorptive phase. No such phase exists in our case (at the tree level), so we expect the C_1 terms to vanish for neutral Higgs decays; (5) show explicitly that this is indeed the case. The situation is a bit more complicated for the decays of charged Higgs bosons, since here the initial and final states are not CP self-conjugate. However, since spin correlations can only be measured if both $\tilde{\chi}$ states produce visible decay products, while $H^- \to \tilde{\chi}^-\tilde{\chi}^0$ decays are dominated by the $\tilde{\chi}_1^-\tilde{\chi}_1^0$ final state, we will only discuss spin correlations for

[4] Note that diagonal $H_i^0H_i^0$ production remains forbidden at e^+e^- colliders even in the presence of CP violation, due to the Bose symmetry of the final state

the "completely visible" decays of the heavy neutral Higgs bosons, into either $\tilde{\chi}_1^{\pm}$ or $\tilde{\chi}_2^0$ pairs.

We can construct three polarization asymmetries from the spins of the final $\tilde{\chi}$ states,

$$\mathcal{A}_a^{ij} = \frac{C_a^{ij}}{C_0^{ij}} \quad [a = 1, 2, 3]. \tag{11}$$

We already saw that $\mathcal{A}_1^{ij}$ is forced to vanish, but the other CP odd asymmetry $\mathcal{A}_3^{ij}$ is allowed. The polarization asymmetry $\mathcal{A}_2^{ij}$ is CP even for neutral Higgs boson decays, but can yield additional information about the phases. The statistical error with which an asymmetry can be measured is proportional to the square root of the number of events in the sample. The significance with which an asymmetry can be established experimentally is therefore determined by an effective asymmetry, defined in terms of the coefficients C_a^{ij} in (10) as

$$\hat{\mathcal{A}}_a^{ij} = \mathcal{A}_a^{ij} \sqrt{\mathcal{B}(H \to \tilde{\chi}_i \tilde{\chi}_j)}. \tag{12}$$

For perfect detection efficiency and polarization analyzing power, the number of Higgs bosons required for detecting the asymmetry at $1-\sigma$ level is then simply given by $\hat{A}^{-2}$. The phase space distribution of $\tilde{\chi}$ decay products only yields information about the $\tilde{\chi}$ spin if the left- and right-handed $\tilde{\chi}$ couplings describing this decay are different. This is generally true for $\tilde{\chi}_1^{\pm}$ decays, so we expect the analyzing power for $\tilde{\chi}_1^- \tilde{\chi}_1^+$ final states to be usually fairly large, in the tens of percent range at least. On the other hand, the couplings of $\tilde{\chi}_2^0$ to both neutral Higgs and neutral gauge bosons give L and R couplings of equal magnitude. The analyzing power of the $\tilde{\chi}_2^0 \tilde{\chi}_2^0$ final states will therefore be very small unless sfermion exchange contributions are significant. In scenarios with heavy first- and second-generation sfermions this might still be the case for $\tilde{\chi}_2^0 \to \tilde{\chi}_1^0 b\bar{b}$ and $\tilde{\chi}_2^0 \to \tilde{\chi}_1^0 \tau^- \tau^+$ decays, which can also have sizable branching ratios. Obviously, more detailed analyses would be required to make more precise statements about the analyzing power, and to estimate the detection efficiencies. Here we simply present results for the effective asymmetries $\hat{\mathcal{A}}$, in order to show that the asymmetries might in fact be large.

In Fig. 4 we show the effective polarization asymmetries $\hat{\mathcal{A}}_{2,3}$ of the "completely visible" supersymmetric heavy Higgs boson decays as functions of Φ_μ for the parameter set (8) with $\Phi_1 = 0$. The left frames are for H_2^0 and the right frames for H_3^0. We see that both asymmetries depend strongly on the phase Φ_μ and their sizes can be significant for a large region of Φ_μ. Note the strong anticorrelations between $\hat{\mathcal{A}}_i(H_2^0)$ and $\hat{\mathcal{A}}_i(H_3^0)$ for both $i = 2$ and $i = 3$, which again results from the different composition of these mass eigenstates in terms of current eigenstates. Recall that these two heavy neutral Higgs bosons are almost degenerate. The mass splitting of 2 to 3 GeV in our case should be sufficient to study H_2^0 and H_3^0 as separate s-channel resonances at a muon collider [4,19]. However, it will be difficult to distinguish decays of H_2^0 and H_3^0

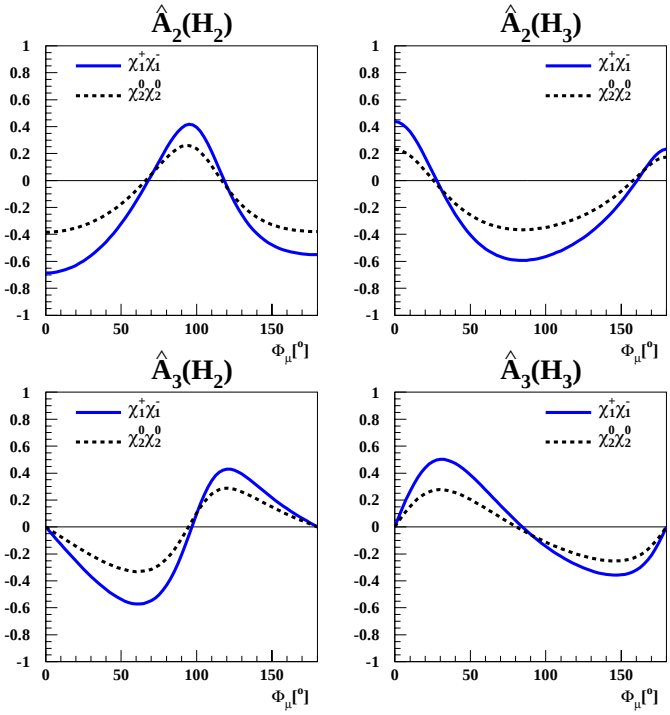

Fig. 4. The polarization asymmetries $\hat{\mathcal{A}}_{2,3}$ in the supersymmetric decays of the heavy Higgs bosons, H_2^0 (*left frames*) and H_3^0 (*right frames*) with respect to the phase Φ_μ. The phases Φ and Φ_1 are set to 0

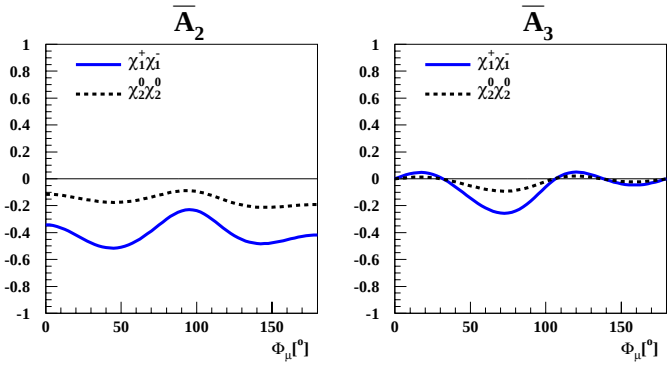

Fig. 5. The average effective polarization asymmetries $\bar{\mathcal{A}}_{2,3}$ in the supersymmetric decays of the heavy Higgs bosons as a function of the phase Φ_μ, for $\Phi = \Phi_1 = 0$

on an event by event basis at e^+e^- colliders; recall that there the dominant production process is $e^+e^- \to H_2^0 H_3^0$, i.e., one produces equal numbers of H_2^0 and H_3^0 bosons. In Fig. 5 we therefore also show the average effective asymmetries, defined as

$$\bar{\mathcal{A}}_i \equiv \frac{\mathcal{A}_i(H_2^0)B(H_2^0 \to \tilde{\chi}\tilde{\chi}) + \mathcal{A}_i(H_3^0)B(H_3^0 \to \tilde{\chi}\tilde{\chi})}{\sqrt{B(H_2^0 \to \tilde{\chi}\tilde{\chi}) + B(H_3^0 \to \tilde{\chi}\tilde{\chi})}}. \tag{13}$$

We see that averaging in this manner does degrade the asymmetries significantly; nevertheless, the CP violating effective asymmetry might still be of order 20%.

To summarize: we studied Higgs boson decays into charginos and neutralinos in the MSSM with explicit CP

violation. The branching ratios for these decays are sizable whenever the Higgs boson mass exceeds the sum of gaugino and higgsino masses or whenever there is significant mixing between gauginos and higgsinos in the light $\tilde{\chi}$ states, provided $\tan\beta$ is not very large. We found that some of these branching ratios depend significantly on the CP violating phases. Much of this sensitivity comes from the dependence of neutralino and chargino masses on these phases; these masses can more easily be measured in the direct production and decay of charginos and neutralinos. However, the Dirac structure of the relevant coupling (scalar and/or pseudoscalar) also plays an important role, and is directly related to CP violation. Moreover, we found that correlations between the spins of the $\tilde{\chi}$ states produced in the decays of heavy neutral Higgs bosons can lead to large asymmetries, one of which is nonzero only in the presence of CP violation. This is true even in the absence of CP violating mixing between the neutral Higgs bosons, and could thus signal "direct" CP violation in Higgs boson decays. We hope that this result motivates further detailed investigations, which are needed to decide whether these large polarization asymmetries are actually measurable at future colliders.

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